change is small and the shock can be treated as a simple wave to which either eq. (35) or (36) applies. In this approximation the interactions of shock waves and rarefactions can be calculated from eqs. (25) and (26).

3. – Elementary wave interactions.

Equations (32), (34), (35) and (36) uniquely define and limit the values of particle velocity, u, which can be achieved by simple shock or rarefaction from a given state (p_0, V_0, u_0) . This limitation on states which can be reached in a single wave transition supplies a powerful tool for thinking about and calculating the fields of high-amplitude waves. The problem is transformed into a «hodograph » plane in which the variables are (u, p), (u, l), (r, s) or some equivalent set. We shall use u, p here because of continuity conditions on u and p at an interface or boundary. The significance of this choice will appear later.

Various useful representations of a shock and of a rarefaction are shown in Fig. 4. In 4 a) is a cross-section of a half-space to which a pressure p_1 was applied at t=0 and released at $t=t_0$. The pressure profile at this particular $t > t_0$ is shown in 4 b). It consists of a forward-facing shock, designated \mathscr{S}_+ , a region of uniform pressure p_1 and particle velocity u_1 , and a rarefaction \mathcal{R}_+ . The notations \mathscr{S} and \mathcal{R} are introduced here to denote shock and rarefaction waves, respectively. Forward-facing waves are denoted by the subscript * + *, backward-facing by * - *. In Fig. 4 c) the flow is shown in the (x, t) plane. Region I is the uniform initial state (p_0, V_0, u_0) with $u_0 > 0$. The shock front, \mathscr{S}_+ , has constant slope until the following rarefaction overtakes it, reducing its amplitude and velocity. Region II is the uniform state (p_1, u_1, V_1) behind the shock. Region III is the rarefaction \mathscr{R}_+ in which pressure and particle velocity are diminishing. Region IV is again at the ambient pressure p_0 but volume and particle velocity now differ from V_0 and u_0 . The path OAB is the trace of the half-space surface, sometimes called the «piston path», P. The dashed curve is the path of a single particle or mass element traversed successively by \mathscr{S}_+ and \mathscr{R}_+ . Figure 4 d) shows the wave process in the (p, V) plane. The initial shock compression is along the Rayleigh line to the state B on the Hugoniot. The rarefaction, assumed to be isentropic, expands the material along the dashed isentrope to the final state $C(V'_0, p_0, u'_0)$. In Fig. 4 e) the process is shown in the (p, u) plane. The straight line AB with slope $dp/du = \rho_0(D-u_0)$ is the image of the Rayleigh line. The compressed state B lies on the image of the Hugoniot curve and the dashed curve BC is the image of the isentrope of Fig. 4 d). Because the shock process is entropic and because most materials have positive thermal expansion coefficients, the final state (u'_0, p_0) is normally to the left of (u_0, p_0) for forward-facing waves.



Fig. 4. – Forward-facing rarefaction overtaking a shock. a) planes of constant phase in half-space; b) pressure profile, $t > t_0$; c) (x-t) diagram; d) (p-V) diagram; e) $(p \ u)$ -plane.

16